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A CLASS OF MONOTONIC SHOCK-CAPTURING DIFFERENCE SCHEMES. (U)
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20. ABSTRACT (Continue on reverse side if necessary and identify by block number) Two principal approaches used in the numerical study of unsteady inviscid compressible flow problems—"shock-fitting" and "shock-capturing"—are discussed briefly. This paper presents a new example of the latter approach. The proposed method is monotonic and provides second-order accuracy in regions where the concept of order is meaningful. The method is simple and fast running. The monotonicity of a standard finite difference scheme (the well-known MacCormack scheme was chosen in the paper) is achieved by. <div style="text-align: right;">(Continues)</div>		

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20. ABSTRACT (Continued)

means of a local conservative smoothing operation. It eliminated nonphysical ripples, which usually appear near discontinuities when nonmonotonic schemes are used. The method is similar to one developed by Boris and Book and gives essentially the same results, but is much simpler and faster. The proposed method is compared to some other finite difference schemes. Two one-dimensional tests are used: a linear convection equation and a shock tube problem. Finally, the example of nonsteady three-dimensional scattering of a shock wave by a step obstacle of finite width is discussed.

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A CLASS OF MONOTONIC SHOCK-CAPTURING DIFFERENCE SCHEMES

0. Introduction

A broad class of gas-dynamic problems can be treated within the framework of ideal gas theory, i.e., assuming the gas is inviscid and thermally nonconductive. The differential equations of gas dynamics follow from the laws of conservation of mass, momentum, and energy in integral form assuming continuous differentiability of the fluid variables, and constitute a system of quasi-linear hyperbolic equations. It is well known that in general these may possess discontinuous solutions even when smooth initial data are prescribed¹. Physically, the presence of discontinuities in the solution typically signals the appearance of a shock wave. There are two possibilities for correctly describing discontinuous flows within the framework of ideal gas theory. The first consists of breaking up all the regions in the problem into subregions of smooth flow, which are described by the differential equations of gas dynamics, while the discontinuities (boundaries of the subregions), are described by conservation conditions. It is important to distinguish weak discontinuities (discontinuities in derivatives of the fluid variables), tangential discontinuities, and finally, strong discontinuities (shock waves). The second approach consists in utilizing the conservation conditions in integral form, which allows discontinuous solutions. For historical reasons (the differential equations were derived earlier), these are referred to as "generalized," in contrast to the continuous classical equations. We note that consideration of generalized solutions extends the class of possible solutions, and it is therefore necessary to make use of additional considerations in order to determine the correct solution. For example the requirement that entropy not decrease at a discontinuity permits us to exclude

¹Manuscript submitted May 12, 1981.

rarefaction shocks in the flow of a perfect gas (from a formal point of view such a discontinuity is unstable).

In numerical modelling of gas motions in the presence of shock waves both of the above approaches are possible. In the former, the physical jump conditions on the discontinuities dividing the regions of continuous flow are used both in order to obtain the conditions connecting the fluid variables on the two sides of the discontinuity and to determine the motion of the computational mesh points following the discontinuity. Changes in the topology of the lines of discontinuity can be followed either using a moving curvilinear mesh or by means of an algorithm based on a fixed mesh. The first method is most completely worked out in the paper by S. K. Godunov et al.² while the second has been developed by Moretti et al.^{3,4}

The shortcoming of both approaches is the inhomogeneity of the resulting difference schemes and consequent complicated structure of the numerical algorithm. Both approaches, as a rule, demand an a priori knowledge of the flow pattern during the calculation.

When it is impossible to know in advance the flow pattern, or when it is changing qualitatively with time, it is more convenient to make use of "shock capturing" difference schemes, amounting to difference approximations to the integral form of the conservation laws for every computational cell. The form of the difference equations does not depend on the character of the flow or the position of the possible shocks, and therefore such schemes are homogeneous.^{1,5} [Apparently "homogeneous" means universal, i.e., not problem-dependent.--DLB] In this approach shocks appear as regions of abrupt variation of the fluid quantities.

Shock-capturing schemes can also be based on the differential equations. For this purpose small corrections are introduced in the equations, typically of nonlinear form, analogous to physical viscosity. The equations assume parabolic form and permit a smooth solution which tends to the solution of the original system as the "artificial" viscosity vanishes. However, difference analogs of the conservation laws are satisfied in this technique, generally speaking, only approximately. In difference schemes based on the integral form, the mass, momentum, and energy conservation laws are satisfied for every computational cell to roundoff. Such schemes are describable in terms of fluxes across boundaries of the computational cells, and so the conservation laws for each computational region are algebraic consequences of the conservation laws for cells, i.e., the schemes are conservative.⁵

We note that the first approach, as a rule, distinguishes only simple shocks, while the other types of discontinuity are calculated as in the shock-capturing method. A difference scheme employed for solving practical problems is required to be accurate, that is, it must be able to describe flows on comparatively coarse meshes; it must be economical; and it should be simple to employ. The accuracy of the scheme for calculating smooth flows is determined by the order of the approximation. In the neighborhood of a shock, the change in the fluid variables is comparable with their magnitude, so that the concept of the order of the approximation becomes meaningless. It is well known that schemes of the first type lead to smearing of the shock because of the strong numerical viscosity. Schemes of second and higher order give rise to significantly less smearing of shock runs, but are typically

nonmonotonic. One of the principle reasons for the appearance of unphysical oscillations in the neighborhood of a shock is the nonvanishing dispersion of the difference scheme, which leads to misrepresentations of the form and speed of propagation of physical disturbances, especially at short wavelengths. Nonmonotonicity may also be produced by the nonlinearity inherent in real problems.⁶

Artificial viscosity is introduced to suppress unphysical oscillations. Most methods for achieving this, however, do not yield a monotonic scheme, and consequently make the localization of shocks worse and strongly smear large gradients. This problem becomes especially severe in calculations involving strongly shocked flow arising from nonsteady interaction of shock waves with one another and with obstacles.⁷ Recently a number of methods using nonlinear limiters of various sorts to filter unphysical oscillations have appeared.⁷⁻¹⁴ Many of these are highly effective, so that the assertion that "good schemes of first and of higher order smear shock fronts in practically the same degree"² cannot be accepted as correct.

It is convenient to choose an actual difference scheme in several stages. First the dispersion and dissipation properties are studied in the linear approximation using the method of differential approximations or harmonic analysis for the simplest transport equations. It is essential to test the schemes selected through linear analysis on simple one-dimensional problems having an exact solution. The final stage of development of any difference scheme must be its verification on model two-dimensional problems.

In the present paper a nonlinear conservative smoothing procedure is proposed in order to achieve monotonicity in explicit schemes of higher order. Although this smoothing procedure is applicable to arbitrary schemes, the underlying transport algorithm is taken here to be second-order MacCormack differencing.¹⁵ The results of the linear analysis of the dispersion and dissipation properties of this and a number of other widely used schemes are presented in Ref. 16. Below, the second stage in the choice of an optimal scheme is considered in detail. This is conveniently divided into two parts. The first is an investigation of the solution of the linear advection equation, which facilitates an understanding of the nonlinear properties of the scheme. The second part is a calculation of the flow resulting from the evolution of a one-dimensional gas dynamic shock. In conclusion, as an illustration of the possibilities of the proposed numerical method, a three-dimensional problem involving the interaction of a plane shock wave with an obstacle is discussed.

1. Construction of second order monotonic difference schemes

An effective device for constructing monotonic difference schemes having second-order accuracy for smooth flows was presented by Boris and Book.^{11,12} It consists of two stages. In the first a large numerical diffusion is introduced into the solution, which guarantees the monotonicity of the scheme. In the second stage this diffusion is canceled wherever doing so does not introduce new unphysical extrema or accentuate existing ones. By means of this approach a family of FCT (Flux-Corrected Transport) methods was constructed.

We consider the simple example of the Cauchy problem for the linear advection equation

$$\frac{\partial f}{\partial t} + v \frac{\partial f}{\partial x} = 0 \quad (1.1)$$

where

$$f(0, x) = f_0(x).$$

The exact solution has the form $f(t, x) = f_0(x - vt)$. Let f_1^n be the value of the function at the i^{th} grid point of a uniform mesh on the n th time level; let $1 + T$ be the operator which propagates the solution from the n th to the $(n+1)$ st level. The form of T depends on the particular difference scheme used. We define a diffusion operator D by

$$Df_1 = \varphi_{i+1/2} - \varphi_{i-1/2} = Q\delta f_{i+1/2} - Q\delta f_{i-1/2} \quad (1.2)$$

where

$$\delta f_{i+1/2} = f_{i+1} - f_i.$$

Then the first (diffusion) stage can be represented in the form

$$\tilde{f}_i = (1+T+D)f_i^n.$$

Now we cancel the diffusion in such a way that in the resulting solution no new extrema in comparison with \tilde{f}_i appear, and those already present are not amplified. For this purpose in the antidiffusion operator A, given by

$$Af_i = -(\varphi_{i+1/2}^c - \varphi_{i-1/2}^c),$$

we limit the fluxes $\varphi_{i+1/2} = Q\delta f_{i+1/2}$ according to the formula

$$\varphi_{i+1/2}^c = S \cdot \max \{0, \min[S \delta \tilde{f}_{i-1/2}, |\varphi_{i+1/2}|,$$

$$S \delta \tilde{f}_{i+3/2}]\}, \quad (1.3)$$

where $S \equiv \text{sign } \varphi_{i+1/2}$.

The transition from the n th to the $(n+1)$ st time level has the form

$$f_i^{n+1} = (1+A)\tilde{f}_i = (1+A)(1+T+D)f_i^n. \quad (1.4)$$

This technique--the explicit cancellation of the diffusion--leads to retention of some of the diffusion in smooth regions [i.e., where the limiter (1.3) does not operate and $A = -D$], even when $T \equiv 0$. Consequently Boris and Book proposed two other algorithms:

"phoenical"

$$f_1^{n+1} = [(1+A)(1+T) + D]f_1^n \quad (1.5)$$

and

implicit

$$f_1^{n+1} = (1+A)^{-1}(1+T+D)f_1^n, \quad (1.6)$$

which permit complete cancellation of the diffusion when $T \equiv 0$ in smooth regions. We note that using the algorithm (1.4)-(1.6) leads to substantial improvement in the results in comparison with widely used schemes.

In the SHASTA¹¹ scheme the coefficient $Q = 1/8$. Subsequently Boris and Book investigated in detail the influence of the value of magnitude of Q on the dissipative properties of this scheme and determined the optimum dependence of Q on the Courant number.¹² However, practically speaking, this optimization is meaningful only for a linear equation with constant coefficients. For the equations of gas-dynamics, as calculations reveal, good results are obtained for Q between $1/10$ and $1/6$.

One can point out three factors responsible for the success of the method of Boris and Book. First, the diffusion stage, as shown by linear analysis, substantially improves the dispersion properties of the scheme. Second, the diffusion is introduced in a conservative manner, and, finally, it is done so as to permit excellent localization in the neighborhood of the shocks. Evidently it is precisely these last two properties which are most important. In Ref. 13 it was

proposed to interpret the method of Boris and Book as a nonlinear conservative smoothing procedure and to introduce diffusion at the $(n+1)$ st time level according to

$$f_1^{n+1} = (1+A)(1+D)(1+T)f_1^n \quad (1.7)$$

or

$$f_1^{n+1} = (1+A+D)(1+T)f_1^n \quad (1.8)$$

This procedure fails to improve the dispersive properties and falls somewhat short in quality of the results of the algorithms (1.4)-(1.6). However, in contrast to the original method of Boris and Book, which is essentially one-dimensional in character and which in two-dimensional calculations can only be applied via the method of coordinate timestep-splitting of the the difference equations, smoothing in the form of Eqs. (1.7)-(1.8) is easily incorporated in essentially two-dimensional schemes. For this purpose, after each timestep successive smoothing operations are carried out with respect to each of the coordinates. Such smoothing has turned out to be effective in solving a whole series of problems.¹⁷⁻²¹

In the present paper a simpler smoothing procedure is proposed, having a local character while retaining the conservative property of the difference scheme. A constant diffusion is introduced only in regions which are not monotonic, i.e., where the solution has a numerical ripple. As a result of this, just as in the method of Boris and Book, there is a certain amount of smearing of physical extrema, but for the most part only oscillations produced by the numerical scheme are

removed. The conservative property is assured by introducing the diffusion in the form of fluxes across the cell boundaries. Using the notation introduced above, we can write the smoothing at the n th time level as

$$f_1^{n+1} = (1+T)f_1^n + D^* f_1^n, \quad (1.9)$$

and on the $(n+1)$ st time level in the form

$$f_1^{n+1} = (1+D^*)(1+T)f_1^n. \quad (1.10)$$

In these expressions

$$D^* f_1 = \varphi_{i+1/2}^* - \varphi_{i-1/2}^*,$$

where

$$\varphi_{i+1/2}^* = \begin{cases} \varphi_{i+1/2}, & \delta f_{i+1/2} \delta f_{i+3/2} < 0 \text{ or } \delta f_{i+1/2} \delta f_{i-1/2} < 0 \\ 0 & \text{otherwise.} \end{cases} \quad (1.11)$$

In order to code this prescription it is convenient to search the entire array f_i , recording the boundaries across which the flux is nonvanishing, and then in a second pass calculate the nonzero fluxes.

This procedure for introducing artificial viscosity runs substantially faster than the method of Boris and Book, while yielding comparable results. Note that, in contrast with the algorithms (1.4)-(1.7), smoothing according to the prescription (1.8)-(1.10) in regions where the fluid quantities vary monotonically in general leaves the solution unchanged.

2. Solution of the linear transport equation

In this and the following section the proposed numerical method is compared with a number of different schemes, using a one dimensional model problem having an exact solution. The schemes of S. T. Godunov²², MacCormack¹⁵, V. V. Eremin and Yu. M. Lipnitskii²³, of first, second and third order, respectively, are considered and also the non-linear scheme of Boris and Book, SHASTX¹², embodying the algorithm (1.5). These schemes are widely employed and have performed well in practice; therefore the present analysis permits one to carry out an objective evaluation of the proposed method.

Let us approximate equation (1.1) on a uniform mesh with $\Delta x = 1$. The computational mesh contains a total of 100 cells. On the boundaries periodicity conditions are imposed. The initial conditions are given in the form

$$f(0, x) = \begin{cases} 0.5, & x > L \\ \varphi(x), & 1 \leq x \leq L \end{cases}$$

where $L = 20$ and

$$\varphi(x) = 2 \tag{2.1}$$

or

$$\varphi(x) = 0.5 + 0.075x. \tag{2.2}$$

In Fig. 1 are shown the results of the calculations after 800 timesteps with Courant number $v = 0.2$, i.e. for a total running time $\bar{t} = Vt/L$, after which the initial square wave (2.1) of width 20 cells has been carried across 160 cells. The curves (a1), (a2), (a3), correspond to the schemes of Godunov (which coincides in the linear case with one-sided differencing), MacCormack (equivalent to the scheme of Lax and Wendroff), and Eremin-Lipnitskii; (b) to the SHASTX scheme; (c),

(d), (e) correspond to the monotonic MacCormack scheme with smoothing according to algorithms (1.7), (1.10), and (1.9). The best results were obtained with SHASTX. The SHASTA¹¹ algorithm and the monotonic MacCormack scheme with the smoothing (1.9) were very slightly inferior to this. The smoothing procedures (1.7) and (1.10) give worse results, which are comparable, however, with the results obtained from the third-order scheme. The first-order and second-order schemes yielded unsatisfactory approximations to the solution.

Since the solutions of the linear advection equation using the algorithms (1.7) and (1.10) were inferior, however slightly, to the algorithm (1.9), in the remainder of this section we will present the results of calculations carried out using the latter. We must point out, however, that these other algorithms will be analyzed below, inasmuch as they possess definite advantages in solving real gas-dynamic problems. The results of the present section are applicable in problems where linear equations are solved, for example, in certain meteorological problems.

Figure 2 illustrates the "flexibility" of the various schemes in the sense of Ref. 2, i.e., the capability of solving problems in which the Courant number varies greatly over the computational region. Here profiles are shown at $\bar{t} = 3$, obtained for Courant number $v = 0.6$ (curve 1) and $v = 0.2$ (curve 2) using first-order (a), second-order (b), third-order (c) and the monotonic MacCormack (d) schemes. The superiority of the latter is indubitable. The calculation was not carried out with SHASTX, which requires that condition $v < 0.5$ be satisfied¹¹.

As we remarked in Section 1, the smoothing which removes numerical

oscillations smears out physical extrema. Consequently, the test problem using initial conditions in the form of a triangular profile (2.2) constitutes a stringent test of a scheme. Figure 3 shows profiles at time $\bar{t} = 1$ (curves 1) and $\bar{t} = 20$ (curves 2) for the same schemes as in Fig. 2, if $v = 0.2$. The first-order scheme practically "forgets" the initial conditions; the second-order scheme qualitatively changes the shape of the pulse as time elapses, while the third-order scheme preserves the "height" of the disturbance well. The monotonic MacCormack scheme is only negligibly inferior to SHASTX (the dashed lines in Fig. 3d).

3. The evolution of a gas-dynamic shock

The problem of the evolution of a gas dynamic shock involves all the characteristic features of one-dimensional ideal gas flow: creation and propagation of a shock wave, contact discontinuity and rarefaction fan. By comparing the results of the calculation with the analytic solution, we can check both the accuracy with which the characteristic flow regions are transported and the speed with which they develop. A comparison of the various schemes mentioned above will be presented, based on this problem.

As was stated earlier, unphysical oscillations in the neighborhood of shocks and steep gradients appear because of the nonvanishing dispersion associated with a difference scheme, the nonlinearity of the gas-dynamic equations, and also for several other reasons²⁴. Besides this, it is possible that the various nonlinear smoothing operators may interact with one another nonselfconsistently.

The system of equations has the following form:

$$\int_{x_1}^{x_2} \rho \Big|_{t_1}^{t_2} dx + \int_{t_1}^{t_2} \rho u \Big|_{x_1}^{x_2} dt = 0,$$

$$\int_{x_1}^{x_2} \rho u \Big|_{t_1}^{t_2} dx + \int_{t_1}^{t_2} (\rho u^2 + p) \Big|_{x_1}^{x_2} dt = 0.$$

$$\int_{x_1}^{x_2} e \Big|_{t_1}^{t_2} dx + \int_{t_1}^{t_2} (e + p) u \Big|_{x_1}^{x_2} dt = 0.$$

Here $e = p/(\gamma-1) + \rho u^2/2$, where γ is the adiabatic index. We consider the evolution of a shock with initial pressure ratio $p_2/p_1 = 2$ and density ratio $\rho_2/\rho_1 = 1$, and $\gamma = 1.4$. In Fig. 4 are shown density

profiles obtained using the Godunov (a), Eremin-Lipnitskii (b), MacCormack (c) and SHASTX (d) schemes. The exact solution is indicated by the heavy continuous line and the numerical results up to $t = 25$ by a fine continuous line. The first-order scheme, as one might expect, badly smears the shock front and contact discontinuity and represents the rarefaction fan poorly. The Eremin-Lipnitskii and MacCormack schemes are nonmonotonic and yield oscillations in the neighborhood of the shocks. SHASTX, which represents the shock front very well, gives rise to small undershoots in the density in the neighborhood of the contact discontinuity and rarefaction fan. For this case the SHASTA scheme, as described by Boris and Book, gives practically the same results as does SHASTX. All of the schemes accurately represent the pressure profile between the shock wave and the rarefaction fan.

In Fig. 5 are shown results of calculations using the monotonic MacCormack scheme with various forms of smoothing. Fig. 5a corresponds to algorithm (1.7). One shortcoming of this approach lies in the presence of undershoots in density in the neighborhood of the shocks and rarefaction fans. Very similar results are obtained when one applies the smoothing (1.9) to all of the equations in system (3.1) (Fig. 5b).

In the present work it is proposed to carry out self-consistent smoothing of the density, momentum, and energy in regions where nonmonotonicity in the density profile exists, using the procedures (1.9) and (1.10) described above. The justification for this lies in the fact that every stable one-dimensional shock has a density discontinuity. In particular, this self-consistent smoothing introduces no changes in the velocity at a contact discontinuity, since

the density and momentum are smoothed in a "related" manner. It also somewhat reduces the expenditure of machine time, since in order to determine nonmonotonicity, one array of numbers instead of three is searched [Cf.(1.11)]. We note that this type of smoothing cannot be implemented using algorithms (1.4) - (1.8) because of their two-stage character.

The density profiles obtained using self-consistent smoothing at the n^{th} and $(n+1)^{\text{st}}$ levels are shown in Figs. 5c and 5d. These results are clearly superior to all those shown previously. It is interesting to observe that the difference in the results obtained by smoothing according to the algorithms (1.9) and 1.10) is much smaller than in the case of linear advection (Fig. 1). This is indicative of the decisive role played by nonlinear effects in numerical solutions of the gas-dynamic equations.

We pause to draw attention to some peculiarities in the solutions we have found. Most of the calculations were run with $\Delta t = 0.5$. SHASTA, SHASTX, and the MacCormack scheme with self-consistent smoothing, however, yielded a highly oscillatory solution. The oscillations are a consequence of the nonselfconsistency of the smoothing and the interaction between the resulting errors in the various fluid quantities. A separate paper will be devoted to a detailed analysis of the nonlinear properties of the different schemes, the nonmonotonicity resulting from various causes, and ways of contending with the unphysical oscillations that result. We therefore will not go into the reasons for the appearance of computational ripples in detail, but content ourselves with remarking that they vanish as the time step is reduced. The

solutions shown in Fig. 4c and 5b were obtained with a time step $\Delta t = 0.25$.

We now compare the respective running times. Taking as the unit of time that required for a single step according to the MacCormack scheme, the corresponding running times for the other schemes are as follows: monotonic MacCormack with self consistent smoothing, 1.4; with nonselfconsistent smoothing according to algorithm (1.9), 1.5; using algorithm (1.7), 2; for the Godunov scheme 2 - 2.5, depending on how the "large" quantities are calculated; for SHASTA and SHASTX, 3; and for the Eremin-Lipnitskii scheme, 2.5.

We next consider an example in which a strong shock wave is formed. The initial parameters are taken to be the same as in the paper of Boris and Book¹¹ ($p_2/p_1 = 480$, $\rho_2/\rho_1 = 8$, $\gamma = 5/3$, $\Delta t = 0.02$). The Eremin-Lipnitskii and MacCormack schemes in the absence of an additional artificial viscosity do not permit calculation of such a flow. SHASTA and the monotonic MacCormack scheme with smoothing according to algorithm (1.7) give approximately the same results as SHASTX. We therefore make further comparisons of the results obtained, using the method of Godunov, SHASTX, and monotonic MacCormack scheme with self consistent smoothing given by (1.9). In Figs. 6 and 7 are shown density and pressure profiles obtained using these schemes at time $t = 2$ (dashed lines and $t = 4$ (fine continuous lines). A vertical dashed-dotted line is used to show the initial position of the shock. The virtues and shortcomings of the numerical methods under investigation are clearly shown in Table 1, in which the values of density and relative errors at time $t = 4$ are shown. In the left column of the table is shown

the number of computational cells. The Godunov scheme strongly smears the shock front and contact discontinuity and approximates the exact solution poorly in the region of the rarefaction fan. SHASTX is somewhat better than the monotonic MacCormack scheme in representing the shock wave and rarefaction fan, but somewhat inferior to it in representing the contact discontinuity. In addition, SHASTX gives incorrect values in the neighborhood of the initial shock.

The monotonic MacCormack scheme and SHASTX have good dynamical properties. They permit relatively fast determination of the true flow pattern (Cf. the Godunov scheme). This last property is especially important in solving nonstationary problems.

4. Calculation of three dimensional interaction of a shock wave with an obstacle.

The possibilities of the proposed method are illustrated below for the example of reflection of a plane shock wave from a obstacle of finite width. The initial stage of the interaction process is investigated, in which the reflected shock wave begins to form and the pressure loading on the object is a maximum. We remark that this problem is of interest in its own right, inasmuch as experimental determination of nonstationary three-dimensional flows encounters serious difficulties.

The problem is solved in Cartesian coordinates. The mesh is uniform. Timestep splitting was employed in solving the difference equations^{25,26} which permitted a substantial saving in running time.

The conservation laws in Eulerian coordinates have the form

$$\int_v \rho \Big|_{t_1}^{t_2} dv + \int_{t_1}^{t_2} \oint_s \rho \vec{v} \cdot \vec{n} ds dt = 0,$$

$$\int_v \rho \vec{v} \Big|_{t_1}^{t_2} dv + \int_{t_1}^{t_2} \oint_s (\rho \vec{v} \vec{v} + p) \vec{n} ds dt = 0,$$

$$\int_v e \Big|_{t_1}^{t_2} dv + \int_{t_1}^{t_2} \oint_s (e + p) \vec{v} \cdot \vec{n} ds dt = 0$$

We denote by $L_\alpha (\Delta t)$ the operator which performs a step in the direction α :

$$f_{ijk}^{n+1/3} = L_\alpha (\Delta t) f_{ijk}^\alpha.$$

L_α is defined in the following manner:

$$\tilde{f}_{ijk} = f_{ijk}^n - \frac{\Delta t}{\Delta h} (F_{i+q, j+n, k+s}^n - F_{ijk}^n),$$

$$f_{ijk}^{n+1/3} = \frac{1}{2} [f_{ijk}^n + \tilde{f}_{ijk} - \frac{\Delta t}{\Delta h} (\tilde{F}_{ijk} - \tilde{F}_{i-q, j-r, k-s})],$$

where

$$f = \begin{pmatrix} \rho \\ \rho u_x \\ \rho u_y \\ \rho u_z \\ e \end{pmatrix}, \quad F = \begin{pmatrix} \rho \\ \rho u_x \\ \rho u_y \\ \rho u_z \\ e + p \end{pmatrix} u_\alpha + \begin{pmatrix} 0 \\ q \\ r \\ s \\ 0 \end{pmatrix} p,$$

$$q = \delta_{x\alpha}, \quad r = \delta_{y\alpha}, \quad s = \delta_{z\alpha}.$$

The smoothing is carried out according to the algorithm (1.9). The transverse momentum components were smoothed independently.

The complete transport operator will look as follows:

$$L(2\Delta t) = L_y(\Delta t) L_x(\Delta t) L_z(\Delta t) L_x(\Delta t) L_y(\Delta t).$$

Written thus, even if the split operators $L_\alpha(\Delta t)$ do not commute, the difference scheme has second-order accuracy over a double timestep²⁷.

The stability condition for the split equations is

$$\Delta t \leq \min_{\alpha} (\Delta t_{\alpha}), \quad \alpha = x, y, z,$$

where

$$\Delta t_{\alpha} \leq \min_{ijk} \left(\frac{\Delta h}{u_{\alpha} + a} \right),$$

and a is the speed of sound. This condition turns out to be considerably less stringent than for the unsplit operators:

$$\Delta t \leq \min \left(\frac{\Delta h}{|\vec{V}| + a\sqrt{3}} \right).$$

Since Δt_x (x is the direction of propagation of the shock wave) is usually much smaller than Δt_y and Δt_z , it makes sense to carry out the calculation in the y and z directions with a double timestep. Thus using

$$L_\alpha(2\Delta t) = L_\alpha(\Delta t) L_\alpha(\Delta t), \alpha = y, z,$$

we finally obtain

$$f_{ijk}^{n+2m\Delta t} = L_y(\Delta t) [L_x(\Delta t) L_z(2\Delta t) L_x(\Delta t) L_y(2\Delta t)]^{m-1} \cdot L_x(\Delta t) L_z(2\Delta t) L_x(\Delta t) L_y(\Delta t) f_{ijk}^n.$$

Use of this algorithm instead of

$$f_{ijk}^{n+2m\Delta t} = L^m(2\Delta t) f_{ijk}^n$$

permits reduction of the computational volume by a factor of 1.5.

In the test the computational mesh consisted of $20 \times 20 \times 15$ cells. Calculation of a single case up to $\bar{t} = 1.2 - 1.5$ required about 2 hours of time on the M4030. Here $\bar{t} = tV_{amp}/h$, where V_{amp} is the speed of the reflected shock wave which arose as a result of the interaction of the incident wave with the square object, and h is the step height.

A typical flow pattern is shown in Fig. 8. At $\bar{t} = 1.1$ the isobars are shown in (a) with separation $\Delta p = 10.0$, and the lines of constant Mach number (b) at intervals $\Delta M = 0.2$. The Mach number of the incident shock wave $M_x = 5.0$ and the ratio of width to height of the step was $l/h = 5.2$. The adiabatic index was $\gamma = 1.4$ and the pressure in the

undisturbed gas was $P_0 = 1.0$. It is clear that the method permits excellent resolution of the shock wave and the flow in the neighborhood of the corner points.

In Fig. 9 are shown the isobars in two cross sections $x = \text{const}$, located equidistantly and parallel to the boundary of the step at a distance $0.3 h$. The ratio $l/h = 2.0$ in (a), 3.6 in (b), and 5.2 in (c). In Fig. 9a are shown the isobars for the plane case ($l/h = \infty$).

Figure 10 illustrates the influence of the ratio l/h on the bending of the shock wave in the xy plane. Curves 1,2,3 correspond respectively to $l/h = 2.0, 3.6$, and 5.2 (the bending is related to the bending for the plane case).

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Table 1

[illegible]

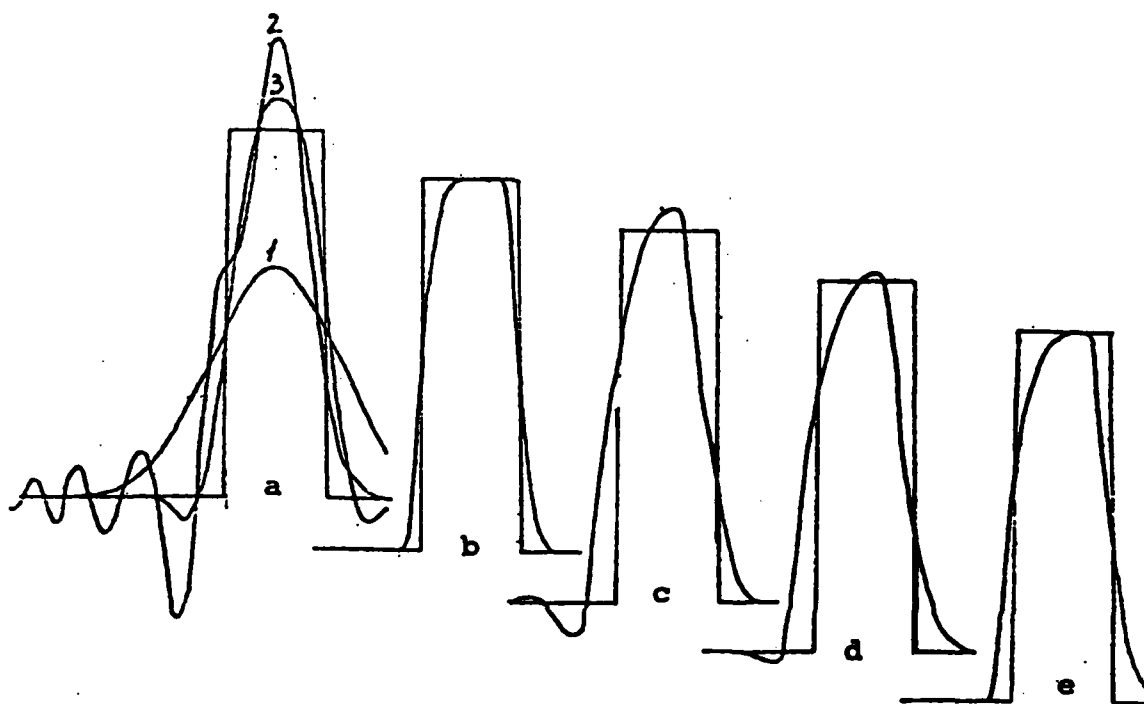


Figure 1

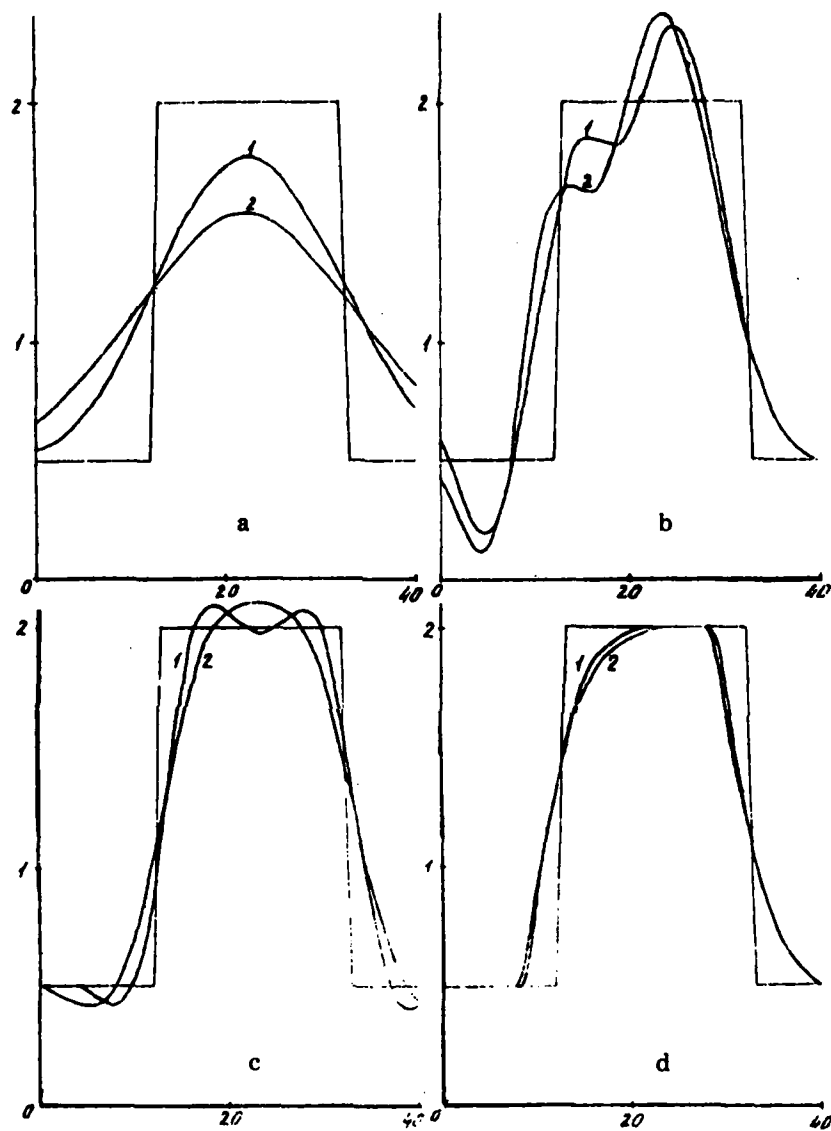


Figure 2

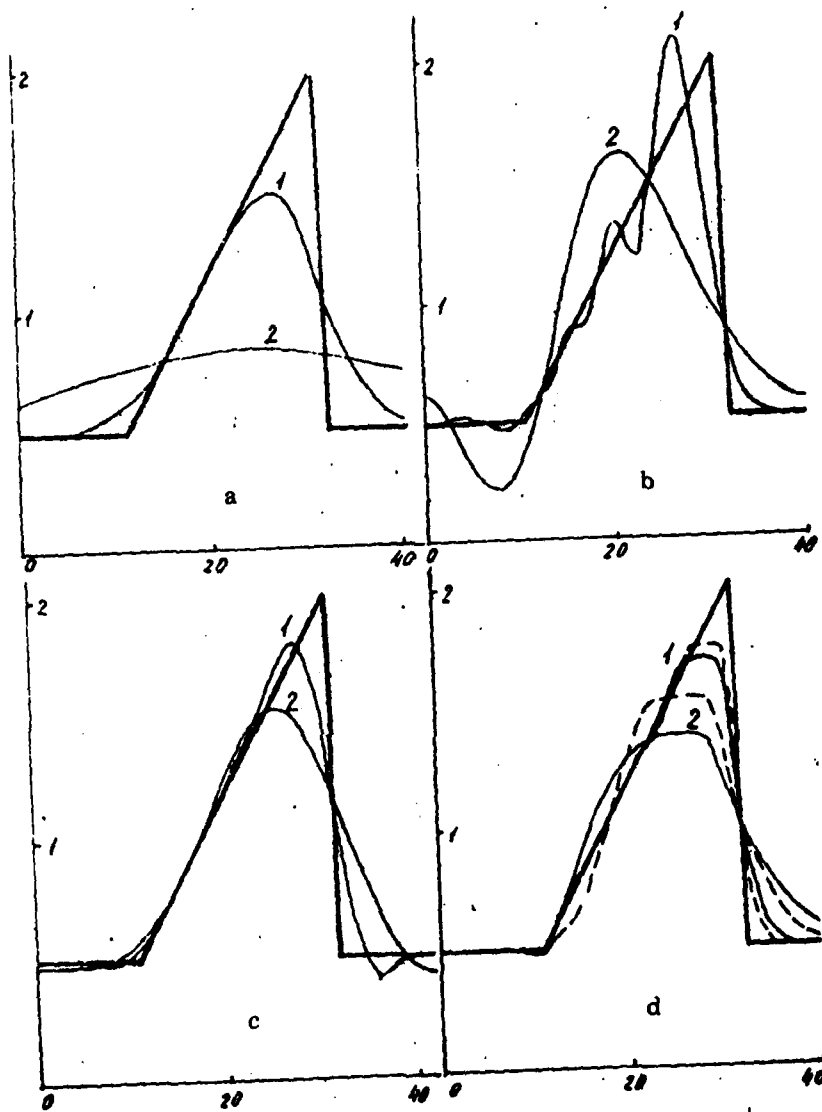


Figure 3

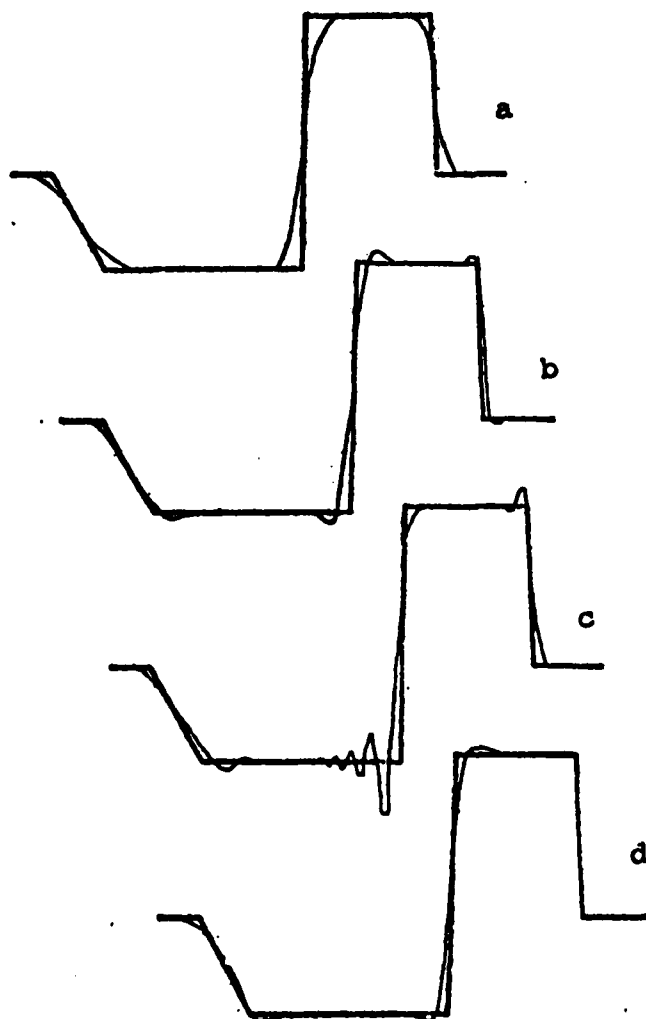


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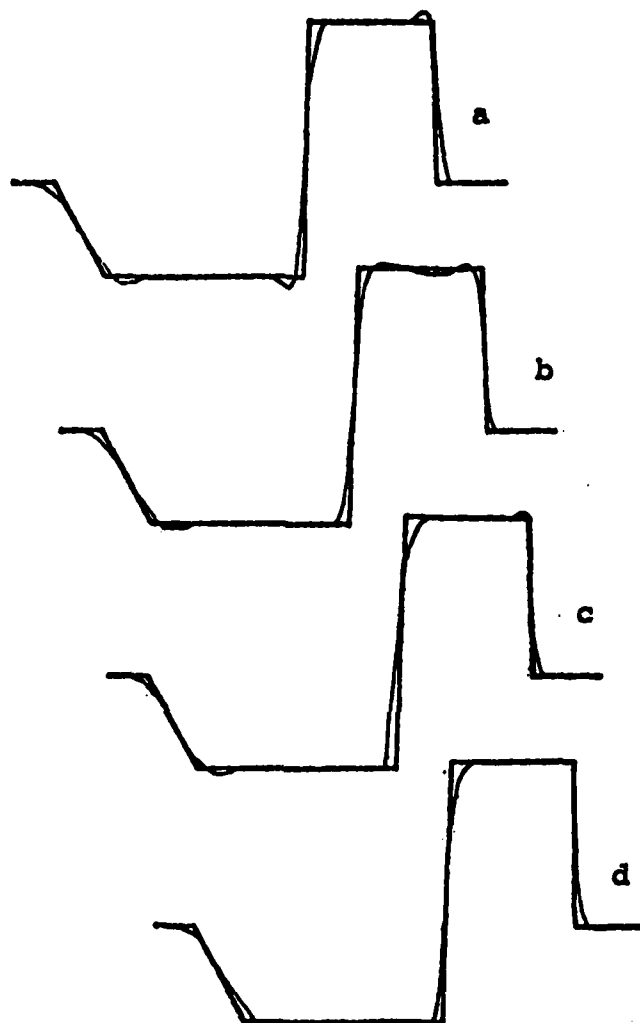


Figure 5

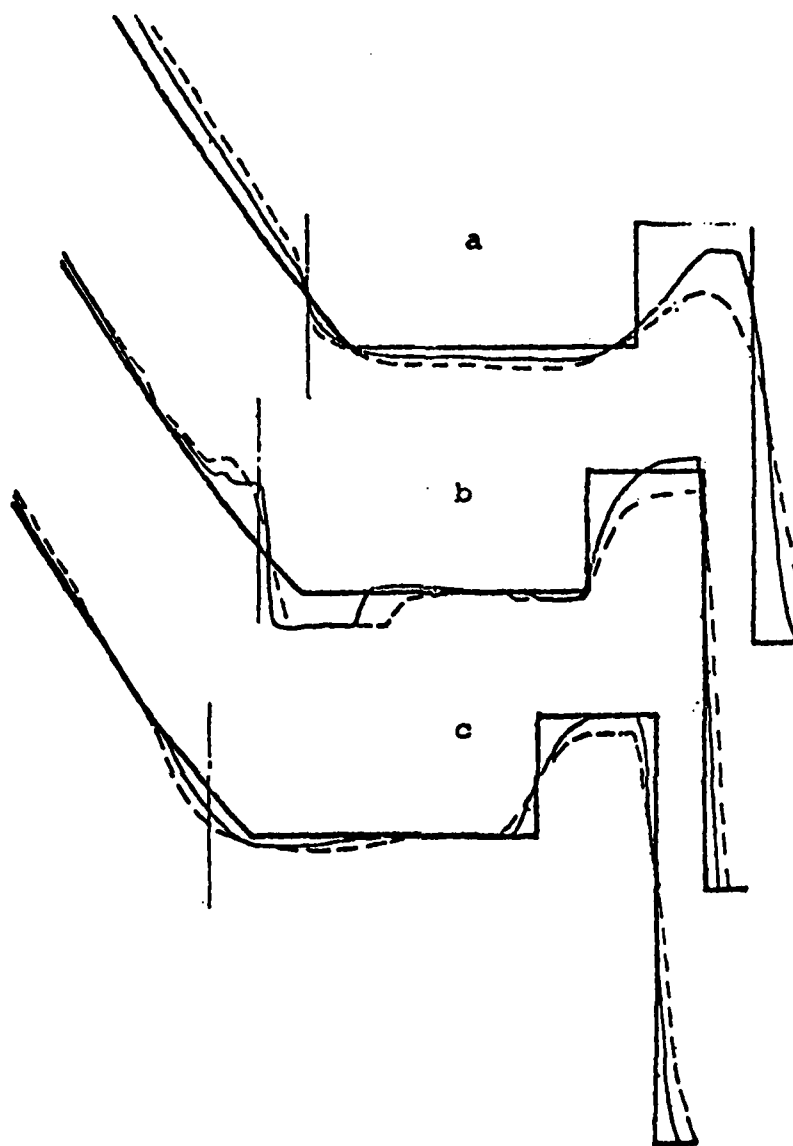


Figure 6

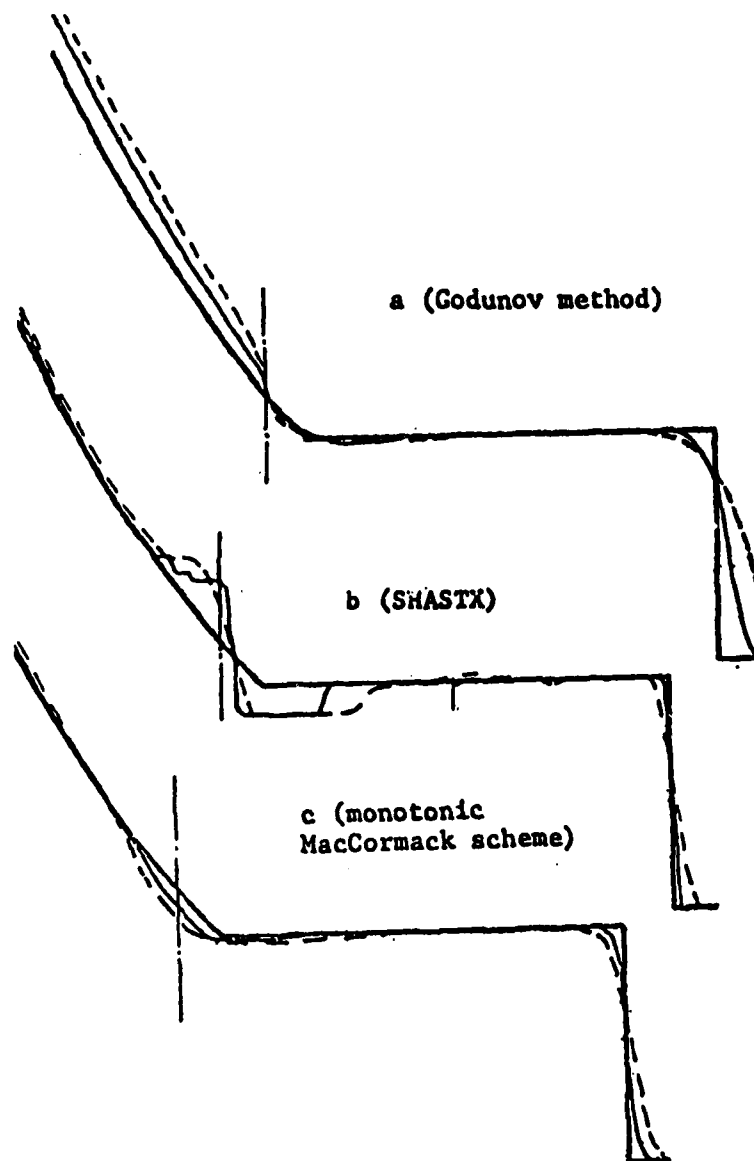


Figure 7

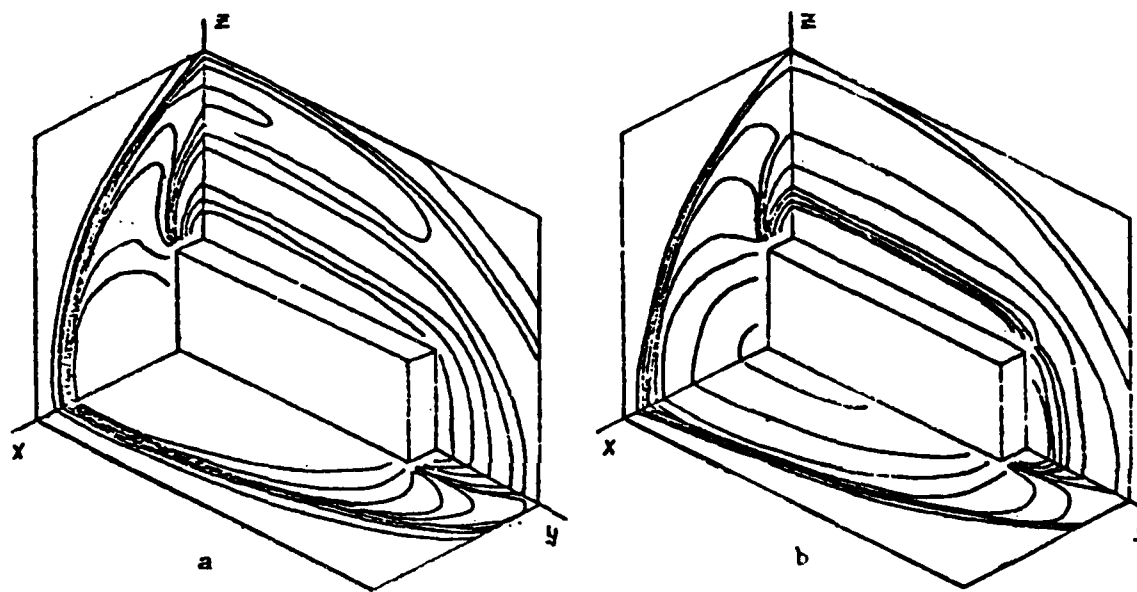


Figure 8

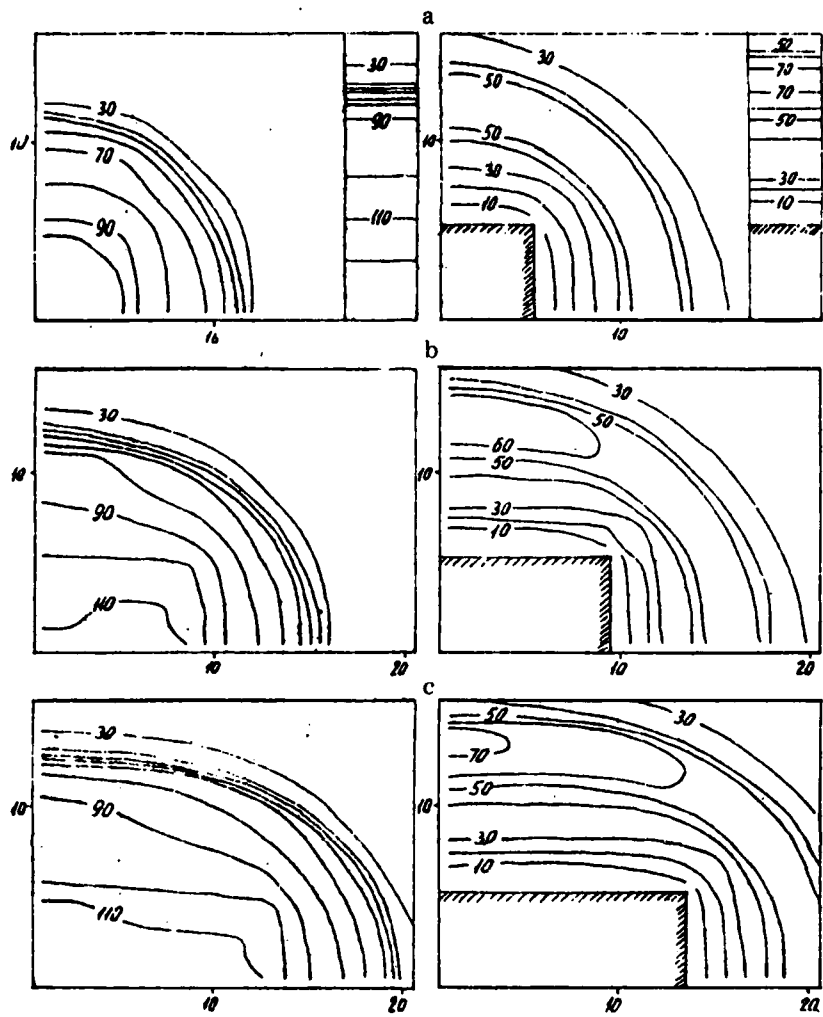


Figure 9

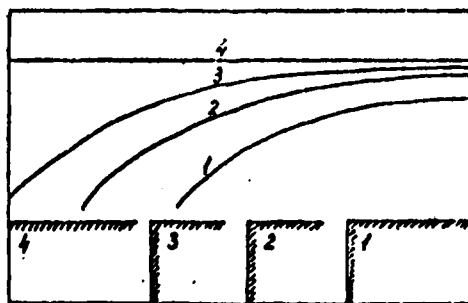


Figure 10

References

- *1. Rozhdestvenskii, B.L. and Yanenko N.N., Systems of Quasilinear Equations and their Applications to Gas Dynamics (Nauka Press, Moscow, 1978).
- *2. Godunov, S.K., Zabrodin, A.V., Ivanov, M.Ya., Krayko, A.N., and Prokopov, G.P., Numerical Solution of Multidimensional Gas-Dynamic Problems (Nauka Press, Moscow, 1976).
- 3. Moretti, G., "Shock Fitting," in Numerical Solution of Hydromechanical Problems (Mir Press, Moscow, 1977), pp. 55-63
- 4. Shankar V., Rutler P., Anderson D.A. Diffraction of a shock wave by a compression corner: Part II - single Mach reflection. AIAA Paper 77-89, 1977.
- *5. Samarskii, A.A. and Popov, Yu. P., Gas-Dynamic Difference Schemes (Nauka Press, Moscow, 1975).
- 6. Lerat A., Peiret R. The Problem of Spurious Oscillations in the Numerical Solutions of the Equations of Gas Dynamics, Lect. Notes in Phys., 35, 251-256 (1975).
- *7. Mikhaylos, A.N., "The Role of Monotonicity in Finite-Difference Schemes for Shock Capturing," J. Comp. Math. and Math. Phys. 17 (14), 1058-1063 (1977).
- *8. Kolgan, V.P., "Use of the Minimal Derivative Principle in Constructing Finite Difference Schemes for Gas Dynamic Calculations," Scientific Notes of the Central Aerodynamics and Hydrodynamics (?) Institute 3 (6), 68-77 (1972).

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9. B. Van Leer, "Towards the Ultimate Conservative Difference Scheme IV. A New Approach to Numerical Convection," J. Comput. Phys., 23, No. 3 276-299, 1977.
10. Forester C.K., "Higher order monotonic convective difference schemes," J. Comput. Phys., 23 No. 1, 1-22, 1977.
11. Boris J.P., Book D.L., "Flux-Corrected Transport, I. SHASTA, a Fluid Transport Algorithm that works," J. Comput. Phys., 11, No. 1, 38-69, 1973.
12. Book D.L., Boris J.P., Hain K. "Flux-Corrected Transport II. Generalisations of the method". J. Comput. Phys., 18, No. 3, 248-283, 1975.
- *13. Zhmakin, A.I., Popov, F.D., and Fursenko, A.A., "A Smoothing Technique for Calculating Gas Dynamic Shocks," in Algorithms and their Mathematical Applications in Physical Problems (FTI, Leningrad, 1977), v.2, pp. 63-72.
- *14. Kolgan, V.P., "Smoothing Techniques for Higher-Order Difference Schemes", J. Comp. Math and Math. Phys, 18 (5), 1340-1345 (1978).
15. MacCormack R.W., "The Effect of Viscosity in Hypervelocity Impact Cratering," AIAA Paper 69, 1969.
- *16. Voynovich, P.A., Zhmakin, A.I., Popov, F.D., and Fursenko, A.A., "On Calculating Gas Dynamic Shocks", Preprint FTI Acad, Sci. USSR, No. 561 (1977).
17. Voynovich, P.A., Popov, F.D., and Fursenko, A.A., "Numerical Modeling of Shock Wave Reflection From a Concave Curvilinear Surface," Tech. Phys. Lett 4 (6) 313-316 (1978).

- *18. Rudakov, G.M., "Numerical Study of the Nonstationary Interaction of a Supersonic Jet with a Finite Barrier," Preprint V.TS. Sib. Div. Acad. Sci. USSR, Krasnoyarsk (1978).
- *19. Zhukovskii, M.I., Kremenetskii, M.D., Leont'ev, N.V., Novikova, O.I. and Popov, F.D., "Numerical Solution of the Problem of Determining the Spatial Flow in the Final Stages of Powerful Stream Turbines," Teploenergetika 3, 18-22 (1977).
- *20. Zhmakin, A.I., and Fursenko, A.A., "Numerical Investigation of Gas Dynamic Characteristics of MHD Channels" Tech. Phys Lett. 24 (4), 1471-1474 (1973).
- *21. Zhmakin, A.I., and Fursenko, A.A., "Numerical Modeling of the Supersonic Flow of Conducting Gas Interacting with a Magnetic Field." Tech. Phys Lett. 24 (4) 1513-1517 (1978).
- *22. Godunov, D.K., "A Finite-Difference Method for Numerically Calculating Hydrodynamic Shocks," Math Arch. 47 (3), 271-306. (1959)
- *23. Eremin. V.V., and Lipnitskii, Yu.M., "Design of Third-Order - Accurate Multidimensional Difference Schemes", J. Comp Math. and Math. Phys 14 (2), 379-389 (1974).
- 24. Boris J.P., Book D.L. Flux-Corrected Transport. III. Minimal-error FCT algorithms. J. Comput. Phys., 20, No. 4, 397-431, 1976.
- 25. MacCormack R.W., Paullay A.J., "Computational efficiency achieved by time splitting of finite difference operators," AIAA Paper 72-154, 1972.

26. Rizzi, A.V., and Inouye, M., "Finite Volume Method with Time Step-Splitting for Calculating Streamlines around Blunt Bodies", AIAA J. 11, 000 (1973).
- *27. Marchuk, G.I., Methods of Computational Mathematics (Nauka Press, Moscow. 1977).

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